Hong Kong Mathematics Olympiad (1999 – 2000)

Final Event 1 (Group)

香港数学竞赛 (1999 – 2000)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 已知整数 n 除 81849、106392 及 124374 得出的余数相等，求 n 的最大值 a。

Given that when 81849 , 106392 and 124374 are divided by an integer n , the remainders are equal . If a is the maximum value of n , find a .

$a =$

(ii) 设 $x=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ 及 $y=\dfrac{1+\sqrt{3}}{1-\sqrt{3}}$。如果 $b=2x^2-3xy+2y^2$，求 b 的值。

Let $x=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ and $y=\dfrac{1+\sqrt{3}}{1-\sqrt{3}}$. If $b=2x^2-3xy+2y^2$, find the value of b .

$b =$

(iii) 已知 c 为正数，如果只有一条直线穿过点 $A(1, c)$ 且与曲线 C : $x^2+y^2-2x-2y-7=0$ 相交于一点，求 c 的值。

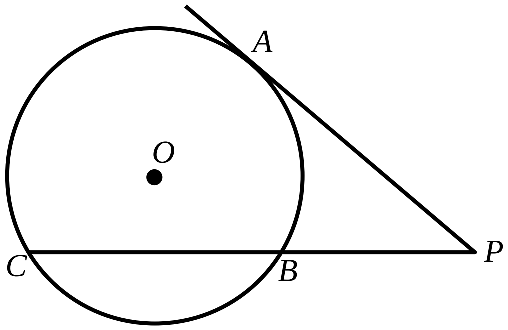
Given that c is a positive number . If there is only one straight line which passes through point $A(1, c)$ and meets the curve C : $x^2+y^2-2x-2y-7=0$ at only one point, find the value of c .

$c =$

(iv) 在图一， PA 切圆于 A， O 为圆心。如果 $PA = 6$， $BC = 9$， $PB = d$， 求 d 的值。

In Figure 1, PA touches the circle with center O at A. If $PA = 6$, $BC = 9$, $PB = d$, find the value of d.

$d =$



图一

Figure 1

Hong Kong Mathematics Olympiad (1999 – 2000)

Final Event 2 (Group)

香港数学竞赛 (1999 – 2000)

决赛项目 2 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 如果 191 为两个连续平方数之差，而 a 为其中最小的平方数，求 a 的值。

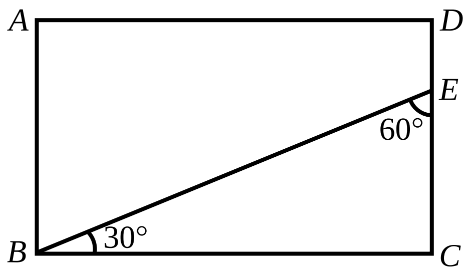
If 191 is the difference of two consecutive perfect squares, find the value of the smallest square number, a.

$a =$

(ii) 在图二(a)，$ABCD$ 是一长方形。$DE : EC = 1 : 5$，且 $DE = 12^{\frac{1}{4}}$。ΔBCE 沿 BE 折去另一方。设 b 为图二(b)中阴影部份的面积，求 b 的值。

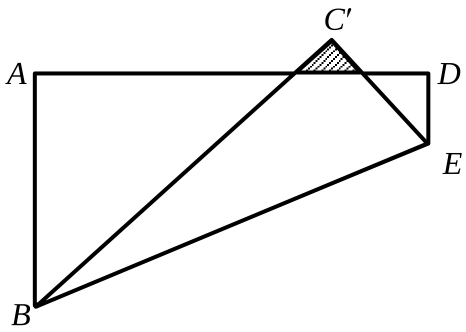
In Figure 2(a), $ABCD$ is a rectangle. $DE : EC = 1 : 5$, and $DE = 12^{\frac{1}{4}}$. ΔBCE is folded along the side BE. If b is the area of the shaded part as shown in Figure 2(b), find the value of b.

$b =$



图二(a)

Figure 2(a)



图二(b)

Figure 2(b)

(iii) 设曲线 $y=x^2-7x+12$ 与 x 轴的交点为 A 及 B，而与 y 轴的交点为 C。如果 c 是ΔABC 的面积，求 c 的值。

Let the curve $y=x^2-7x+12$ intersect the x-axis at points A and B, and intersect the y-axis at C. If c is the area of ΔABC, find the value of c.

$c=$

(iv) 设 $f(x)=41x^2-4x+4$，$g(x)=-2x^2+x$。如果 $f(x)+kg(x)=0$ 只有一个根，求 k 的最小值 d。

Let $f(x)=41x^2-4x+4$ and $g(x)=-2x^2+x$. If d is the smallest value of k such that $f(x)+kg(x)=0$ has a single root, find d.

$d=$

Hong Kong Mathematics Olympiad (1999 – 2000)

Final Event 3 (Group)

香港数学竞赛 **(1999 – 2000)**

决赛项目 **3 (**团体**)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

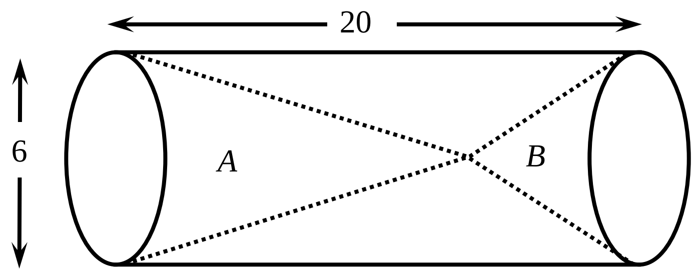
(i) 设 $a=\sqrt{1997\times1998\times1999\times2000+1}$，求 a 的值。

Let $a=\sqrt{1997\times1998\times1999\times2000+1}$, find the value of a.

$a=$

(ii) 在图三，圆管的长为 20 及直径为 6，内有两个圆锥体 A 和 B。A 及 B 的体积比例为 $3:1$。如果 b 是 B 的高度，求 b 的值。

In Figure 3, A and B are two cones inside a cylindrical tube with length of 20 and diameter of 6. If the volumes of A and B are in the ratio $3:1$ and b is the height of the cone B, find the value of b.



图三

Figure 3

$b=$

(iii) 现有点 $A\left(\frac{\sqrt{10}}{2},\frac{\sqrt{10}}{2}\right)$ 和圆 $C: x^2+y^2=1$。如果 c 是通过点 A 与圆相切直线的最大斜率，求 c 的值。

If c is the largest slope of the tangents from the point $A\left(\frac{\sqrt{10}}{2},\frac{\sqrt{10}}{2}\right)$ to the circle $C: x^2+y^2=1$, find the value of c.

$c=$

(iv) 在坐标平面的原点上有一点 P。假如掷出骰子的点数 n 是偶数，P 在 x 方

向右前进 n；如果 n 是奇数，P 在 y 方向上前进 n。如果有 d 种不同掷法使得 P 到达点 $(4, 4)$，求 d 的值。

P is a point located at the origin of the coordinate plane . When a dice is thrown and the number n shown is even , P moves to the right by n . If n is odd, P moves upward by n . Find the value of d , the total number of tossing sequences for P to move to the point $(4, 4)$.

$d =$

Hong Kong Mathematics Olympiad (1999 – 2000)

Final Event 4 (Group)

香港数学竞赛 (1999 – 2000)

决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 如果 a 是一个三位数，驳在 504 之后，新组成的六位数可被 7 、 9 、 11 整除，求 a 的值。

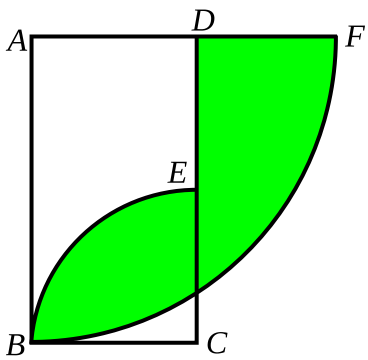
Let a be a 3-digit number. If the 6-digit number formed by putting a at the end of the number 504 is divisible by 7, 9, and 11, find the value of a.

$a =$

(ii) 在图四，$ABCD$ 为长方形，$AB=\sqrt{\dfrac{8+\sqrt{64-\pi^2}}{\pi}}$ ，$BC=\sqrt{\dfrac{8-\sqrt{64-\pi^2}}{\pi}}$ 。BE、BF 分别是以 C 、 A 为圆心的弧。若 b 是阴影部份之面积，求 b 的值。

In Figure 4, $ABCD$ is a rectangle with $AB=\sqrt{\dfrac{8+\sqrt{64-\pi^2}}{\pi}}$ and $BC=\sqrt{\dfrac{8-\sqrt{64-\pi^2}}{\pi}}$. BE and BF are the arcs of circles with centers at C and A respectively. If b is the total area of the shaded parts, find the value of b.

$b =$



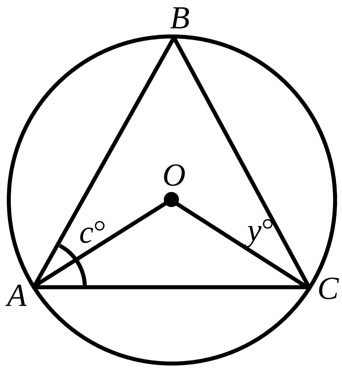
图四

Figure 4

(iii) 在图五， O 为圆心， $c° = 2y°$， 求 c 的值。

In Figure 5, O is the centre of the circle and $c° = 2y°$. Find the value of c.

$c =$



图五

Figure 5

(iv) A、 B、 C、 D、 E、 F、 G 七个人围圆桌而坐。如果 B 及 G 都与 C 相邻而坐的坐法总数为 d， 求 d 的值。

A, B, C, D, E, F, G are seven people sitting around a circular table. If d is the total number of ways that B and G must sit next to C, find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1999 – 2000)

Final Event 5 (Group)

香港数学竞赛 (1999 – 2000)

决赛项目 5 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 如果 a 是可被 810 整除的最小立方数，求 a 的值。

If a is the smallest cubic number divisible by 810, find the value of a.

$a =$

(ii) 设 b 是函数 $y=\left|x^2-4\right|-6x$ (其中 $-2\le x\le 5$) 的最大值，求 b 的值。

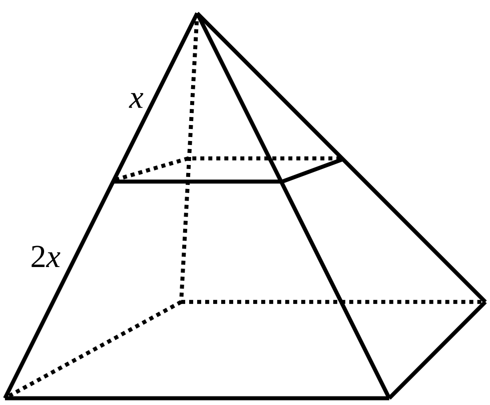
Let b be the maximum of the function $y=\left|x^2-4\right|-6x$ (where $-2\le x\le 5$), find the value of b.

$b =$

(iii) 图六为一个正方形底的锥体。若从底部向上并在 $\frac{2}{3}$ 之高度平行横切，并设 $1:c$ 为上面细锥与余下底部体积的比，求 c 的值。

In Figure 6, a square-based pyramid is cut into two shapes by a cut running parallel to the base and made $\frac{2}{3}$ of the way up. Let $1:c$ be the ratio of the volume of the small pyramid to that of the truncated base, find the value of c.

$c =$



图六
Figure 6

(iv) 如果 $\cos^6\theta+\sin^6\theta=0.4$，及 $d=2+5\cos^2\theta\sin^2\theta$，求 d 的值。

If $\cos^6\theta+\sin^6\theta=0.4$ and $d=2+5\cos^2\theta\sin^2\theta$, find the value of d.

$d=$